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FUNDAMENTALS OF QUAN-TITATIVE ANALYSIS

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Dedicated to my students.

1 Probability and Probability Distribution

1.1 Probability - Introductory Remarks

Probability, as anything else in mathematics, has a very intuitive meaning and a very complicated definition. The latter is eminent for a thorough understanding of probability. This in turn is a necessary condition to understand how a business decision is made towards uncertainty and risk. The intuition suggests that probability is a measure that quantifies the chance of an event to occur. Probability is customarily expressed as a percentage. Therefore, it is defined on a unit interval (no chance would result in 0%, whilst certainty would give 100%) and it cannot take negative values. But why is probability so important to understand the strange ways business is performed? Buyers and sellers do not enjoy full information regarding resources, costs, income, or even specific macroeconomic circumstances they opperate under. And where is an imperfect information, the outcome becomes uncertain. To overcome this obstacle, buyers and sellers collect data on their business practices. (If they state otherwise, they have probably missed the transition to Industry 4.0. Alternatively, they might lie). The inference based on the gathered data approximates a decision pattern, but never result in full certainty. This is why business should be perceived through the lenses of probability. And this is why you need to understand probability.

1.2 Events and σ-algebras

Billy Ray Valentine and Louis Winthorpe III are commodity brokers. Their trade is frozen concentrated orange juice. They buy and sell FCOJ, but not always in this order. Without losing too much generality, we may say that trading FCOJ consists of binary choice phenomena: selling or not selling, buying or not buying. All possible outcomes of an experiment, trading FCOJ in this particular case, define a sample space.

For more information about Ray Valentine, Louis Winthorpe III, Ophelia, Coleman, and Duke & Duke watch 'Trading Places' (1983).

Sample Space: a non-empty set of all possible outcomes of an experiment. Sample space is commonly denoted as Ω.

Events are simply outcomes of experiments and they may form subsets of a sample space Ω. Some events can be decomposed into other events. When Valentine and Whinthorpe use commodity futures contract to sell FCOJ, they deal with a contract size of 15 000 lbs. An event 'selling FCOJ' can be decomposed into 'selling 15 000 lbs of FCOJ', 'selling 30 000 lbs of FCOJ' and so on. Moreover, the decomposition should also consider a price quotation expressed in cents per 1 lbs. Therefore, selling FCOJ is a composite or compound event consisting of more than just one sample points. If an event

Composite or compound events

cannot be decomposed, e.g. granting a 15% discount for order of £1 500, this event is called an elementary or simple event.

Elementary or simple event

Example 1.2.1. — *Discounting Policy as Composite and Elementary Events*.

In an official memorandum, the board of Wernham Hogg Co. advised not to accept quotations with a total discount exceeding 25%. The table presents the discounts granted by regional managers in Swindon and Slough since the memorandum has been issued. The Data Support Department at Wernham Hogg has been asked to summarise the discounting policy in both branches.

Branch:	Discounts granted in %
Swindon	12, 7, 10, 26, 9, 29, 32, 8, 12, 12, 10, 12
Slough	12, 12, 27, 8, 9, 12, 6, 31, 12, 12, 9, 9

To depict the differences between regions, the Data Support Department performed a frequency analysis describing both composite and simple events. The percentage of transactions with total discount exceeding the advised level was equal to 25% and 17% in Swindon and Slough, respectively. An event 'transaction with total discount exceeding the advised level of discounting' is a compound event. The report pointed out that for 38% of all observed transactions, a 12% discount has been granted. An event 'transaction with a total discount of 12%' is a composite event as well: 12% discounts have been approved by both regional managers. In contrast, an event 'transaction with a total discount of 12% in Slough branch' is a simple event.

EVENTS CAN FORM VARIOUS SUBSETS of the sample space Ω. For Vallentine and Whinthorpe, who are active players in the FCOJ market, one possible subset of events is 'sell and buy'. For the discounting policy in Swindon we can be more specific. One of the subsets may consists of 3 discounts: 12%, 7%, and 10%. A collection of subsets of a sample space is called σ-algebra. The simplest σ-algabra is a power set; power set consists of all subsets of a certain set, including the empty set (denoted as $\oslash$).

σ-algebra: a collection of subsets of Ω including an empty set.

Why the empty set? Vallentine and Whinthorpe lived in Philladelphia, but the FCOJ was traded in New York. Had they missed the train to New York, they would not have even entered the trade.

Example 1.2.2. — *σ-algebra and Discounting Policy*.

The board of Wernham-Hogg has accepted the revised discount matrix. The standard discount is now related to the customer classification and the order size. The Uri Omovich Luxury Residence Development, a strategic partner of Wernham Hogg in the Greater London area, has been classified as a Cluster 4 customer. Present the σ-algebras for discounts granted to Omovich Development.

Customer:	Order Size in £1000:			
	up to 2	2-5	5-8	8+
Cluster 1	5%	7%	10%	12%
Cluster 2	7%	10%	12%	15%
Cluster 3	10%	12%	15%	17%
Cluster 4	12%	15%	17%	22%

Omovich Development belongs to the Cluster 4. Before any transaction is closed, the sample space for discounts granted is

$\Omega_{OD} = \{0.12, 0.15, 0.17, 0.22\}$. There are four elements of Ω_{OD}, and the number of sets in F_{OD} is calculated as 2^4 . The σ-algebra consists of $2^4 = 16$ collections of sets (total: 4):

If Ω consists of n elements, then the number of sets in F is equal to 2^n.

1. four one-element sets:

$$\{0.12\}, \{0.15\}, \{0.17\}, \{0.22\},$$

2. six two-element sets (total: 10):

$$\{0.12, 0.15\}, \{0.12, 0.17\}, \{0.12, 0.22\},$$
$$\{0.15, 0.17\}, \{0.15, 0.22\}, \{0.17, 0.22\},$$

3. four three-element sets (total: 14):

$$\{0.12, 0.15, 0.17\}, \{0.12, 0.15, 0.22\},$$
$$\{0.12, 0.17, 0.22\}, \{0.15, 0.17, 0.22\}$$

4. one four-element set (total: 15):

$$\{0.12, 0.15, 0.17, 0.22\}$$

5. and an empty set (total: $2^4 = 16$):

$$\{\oslash\}.$$

Does the collection change once the transactions are closed? Omovich Development is a key customer and always buys for minimum £4000. The 12%-discount is no longer an element of the sample space, hence the collection is reduced to $2^3 = 8$ collections of sets.

WHY DO WE SPEND SO MUCH TIME discussing the events and σ-algebras? The answer is simple - to raise your awareness about what statistics can and cannot solve. If the question is not coherent with the collections of σ-algebras, the questions cannot be answered.
In other words, the explanatory power is limited by the possible combination of events. The board of Wernham-Hogg doesn't ask the data support department what was the probability of buying FCOJ by Omovich Development if the latter has never been involved in the FCOJ trade.

1.3 *Probability Measure*

Probability is a numerical concept, thus in order to define probability there is one component missing - a component that maps the elements of the sample space to the unit interval. In other words, there must be a function assigning a number between 0 and 1 to each and every subset of the ample space.

Example 1.3.1. — *During the last quarter, the Omovich Development has set three orders for the total amount of £22000: two transactions for £9000 and one for £4000. Figure 1.1 depicts the sample space for the orders.*

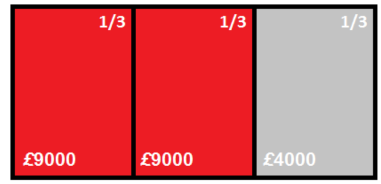

Figure 1.1: Sample Space for OD Orders.

There are three elementary events, and so the sample space is divided into three equal rectangles. Two of them are marked red (£9000) and one grey (£4000). The 'red events' form a union of events, which results in a compound event that we may name an 'order exceeding £5000'. Notice that whether we deal with a union of three elementary events

$$O_{£9000} \cup O_{£9000} \cup O_{£4000} = 1/3 + 1/3 + 1/3 \tag{1.1}$$

or a combination of compound event and elementary one

$$O_{o>£9000} \cup O_{£4000} = 2/3 + 1/3 \tag{1.2}$$

the sums are always equal to 1. Recall now that an empty set belongs to the σ-algebra. The function that we mentioned earlier in this section needs to satisfy two conditions. First, it will assign 0 to the empty set, which intuitively reads if something is impossible to occur, its probability is equal to 0. Second, it will assign 1 to Ω, as we have already proved it by inspecting the unions of events in (1.1) and (1.2). Sometimes the function takes a simple form like equation (1.1), sometimes its form is much more complicated and subtle, but the conditions must be satisfied nonetheless.

Analysing the orders set by Omovich Development, we followed the relative frequency approach to probability. To calculate the probability that Wernham Hogg dealt with an order exceeding £5000, we inspected the frequency with which the orders had occurred during the last quarter. Two out of three have crossed the threshold of £5000. We also utilised the Venn diagrams, a very helpful visual method of depicting various combinations of sets and events.

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